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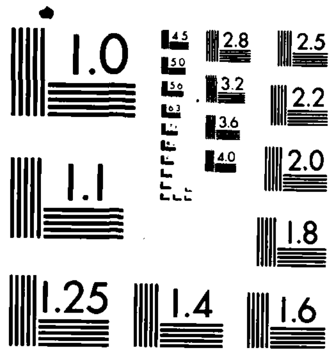

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NONLINEAR DYNAMICS AND CHAOTIC MOTIONS IN FEEDBACK CONTROLLED ELASTIC SYSTEMS

P.J. Holmes
F.C. Moon
R.H. Rand

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ABSTRACT

Local and global bifurcation studies of nonlinear systems subject to linear and nonlinear feedback forces have been completed which have application to robotic devices or controlled elastic structures. Related to these studies has been the application of mathematical knot theory to trace certain bifurcation sequences for two-dimensional maps. This work has led to the conclusion that many other routes to chaos in dynamical systems exist besides period doubling when the map is two-dimensional. The use of computer algebra (MACSYMA) has been developed as a tool to study nonlinear systems. In one application the investigators explored a new control scheme for flexible space structures based on controlling the stiffness matrix. MACSYMA was used along with normal form theory to predict the stability properties of a stiffness controlled system. Other studies using MACSYMA related to problems in robotic dynamics were also completed or started. Finally, experimental work was completed involving the application of mathematics to chaotic motion of flexible structures.

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1. Introduction and Background

The subject of feedback control of elastic structures is an important area in the design of large space structures [1], robots and manipulators [2], and aeroelasticity of flight structures [3]-[5]. Much work has been done on the linear theory of elastic structures with control but less is known about the nonlinear dynamics of such systems.

In the last several years major advances have been made in nonlinear dynamics and new phenomena have been explored with new methods of analysis. Chief amongst them is the subject of chaotic dynamics; that is the appearance of apparantly random motions in a deterministic system [6]. Such behavior is familiar in fluid mechanics, but recently the Cornell Dynamics Group demonstrated similar phenomena in mechanical systems, included buckled beams [7]-[9], systems with bilinear stiffness [10]-[11] and in multi-equilibrium positioning devices such as stepper motors. In the mathematical theory of dynamical systems such phenomena are modelled by strange attractors

The subject of strange attractors and chaos in feedback systems has only recently been addressed--principally because one must include nonlinearities in the system. Sparrow [12], for example, has demonstrated an apparent strange attractor in a single loop piecewise linear feedback system. Mees [13] has also discussed the occurence of chaos in feedback controlled systems. And in a recent unpublished study, the authors have discovered chaos in equations for a feedback controlled elastic positioning device (see discussion below).

The types of systems we are prepared to study can be written in the form:

$$\Sigma\, m_{ij}\ddot{q}_j + \Sigma\, \gamma_{ij}\dot{q}_j + \Sigma\Sigma\, \beta_{ij}\dot{q}_j\dot{q}_k + f_i(q_k) = g_i(\phi_k, U_k(t))$$

$$\Sigma\, c_{ij}\dot{\phi}_j + \Sigma\, a_{ij}\phi_j = h(q_k, \dot{q}_k, U_k(t)) \qquad (1.1)$$

where

q_j are generalized coordinates,

ϕs are feedback control variables or forces,

$U_k(t)$ are prescribed input functions,

and the functions f( ), g( ), and h( ) may be nonlinear functions of their arguments.

These equations describe mechanical systems with feedback forces and include elasticity and gyroscopic forces in order to incorporate rotations in problems of spinning spacecraft as well as robots and manipulators. The equations can also model certain electrical devices such as nonlinear circuits and stepper motors.

Since the inception of this grant in December 1983, we have concentrated on one and two degree of freedom systems in an attempt to gain a basic analytical understanding of "low dimensional" instability and chaos in feedback controlled devices. In the next section we review our progress over the past year and in the final section outline some outstanding questions and our proposed research for the next two years. We list the publications prepared under the present grant in Section 4.

2. Summary of Research Accomplishments: December 1983-November 1984

This summary is divided into three sections, one for each P.I.

2.1 Analytical Studies of Feedback Controlled Oscillators

(P.J. Holmes, S. Wiggins (Grad. Student))

2.1.1 Bifurcation studies. Local and global bifurcation studies of nonlinear oscillators subject to linear and nonlinear feedback have been completed. The systems treated have the form

$$\begin{aligned} &\ddot{x} + f(x,\dot{x},z) = 0 \\ &\dot{z} + g(x,\dot{x},z) = 0 \,. \end{aligned} \tag{2.1}$$

An example is the Duffing equation subject to linear feedback:

$$\begin{aligned} &\ddot{x} + \delta\dot{x} - x + x^3 = -z \\ &\dot{z} + \alpha z = \gamma x \,. \end{aligned} \tag{2.2}$$

The feedback variable, z, is modelled by a differential equation, since we assume inherent dynamics in the control devices (e.g. servomotors with inductance). In [14] and [15] local bifurcation analyses of (2.2) and a fifth order variant of (2.2) were carried out using center manifold theory. See the

reprints attached. In both cases the interaction of steady and periodic motions in complicated "codimension two" bifurcations was described, and the rôle of "planar" homoclinic orbits occurring on the center manifold discussed. The net effect of these phenomena is to promote "ragged" boundaries to the parameter regions in which feedback control is able to stablize the system.

More recently, Wiggins and Holmes [16] have addressed global questions and have developed a variant of Melnikov's method (Guckenheimer and Holmes [17]) to deal with three dimensional perturbations of families of planar Hamiltonian systems. The methods enable us to detect periodic orbits and invariant tori in the three dimensional phase space and to compute their stability and bifurcations in specific examples. We have applied the methods to a number of systems, including modified Duffing and van der Pol equations with feedback control. Used together with "standard" bifurcation methods (Guckenheimer and Holmes [17]), the technique enables one to obtain a fairly complete picture of the dynamics of specific three dimensional systems. A paper describing this work is currently in preparation.

2.1.2 Knot theory and continuation of periodic orbits. P.J. Holmes and R.F. Williams (Northwestern) have used Knot theory, Hamiltonian bifurcation theory, Kneading theory, and symbolic dynamics to trace certain bifurcation sequences for 'quadratic like' two dimensional maps. An example is the Hénon map, which

may be written

$$(x,y) \xrightarrow{F_{\mu,\varepsilon}} (y, -\varepsilon x + \mu - y^2) \ . \qquad (2.3)$$

As the parameters μ and $\varepsilon = \det(DF_{\mu,\varepsilon})$ vary, a Smale horseshoe is created in an uncountable infinity of bifurcations. We have proved that countably many periodic bifurcation sequences of saddle-nodes are reversed as one passes from the one dimensional limit $\varepsilon = 0$ to the area preserving limit $\varepsilon = 1$. This work provides an important insight into the transition to chaos in two dimensional maps and three dimensional flows. It is jointly supported by NSF. A full paper is in preparation [18] and presentations have been made at La Jolla Dynamics Days, January 1984 and the MSRI Berkeley workshop on topology and smooth dynamical systems, June 1984. A paper is to be read at the XVIth International Congress on Theoretical and Applied Mechanics in Lyngby, Denmark, in August [19].

2.2 Analytical and Computer Algebra Analyses of Feedback Systems and Nonlinear Oscillators

(R.H. Rand, W.L. Keith, L.A. Month (Visiting Assistant Professor), T. Chakraborty (Grad. Student))

2.2.1 Normal form and center manifold calculations on MACSYMA

(with W.L. Keith). Paper presented at American Chemical Society Computer Algebra meeting in Philadelphia, August 1984. MACSYMA programs are presented which automatically compute center mani-

folds and normal forms to arbitrary truncation orders (limited only by the memory limitations of the computer). These programs permit calculations to be accomplished which were previously impossible by hand, thereby making these methods more attractive.

2.2.2 Parametric stiffness control of flexible structures (with F.C. Moon). Paper presented at NASA Workshop on Identification and Control of Flexible Space Structures, San Diego, May 1984 [20]. The work involves the system:

$$x'' + (1 + z)x = 0 \quad \text{(stiffness controlled structure)}$$

$$\qquad\qquad (2.4)$$

$$z' = -kz + f(x,x') \quad \text{(feedback law)}$$

We used the MACSYMA program discussed in 1) above to obtain an approximation for the center manifold $z = g(x,y)$ associated with this system. Then we used normal forms to study the flow on the center manifold. Results included the prediction of the stability of the zero solution as well as bifurcation of limit cycles. Results were confirmed by numerical integration.

Note that in this work we had to go to terms of $0(x^5)$ in order to predict the Hopf bifurcations of the limit cycles. This calculation would have been impossible without computer algebra.

2.2.3 Derivation of the Hopf bifurcation formula using Linstedt's perturbation method and MACSYMA. Paper to be presented at American Chemical Society Computer Algebra meeting in

Philadelphia, August 1984 [21]. While the Hopf bifurcation theorem (and the associated formula distinguishing supercritical bifurcations from subcritical) are frequently used by applied researchers, their derivation is less well understood. The idea of this work is to offer a simple derivation based on the well known Lindstedt's perturbation method. The calculation, while straightforward, is lengthy, and the use of MACSYMA greatly simplifies the derivation. This is an example of how computer algebra can be used in proofs.

<u>2.2.4 Dynamics of a system exhibiting the global bifurcation of a limit cycle at infinity</u> (with W. Keith). Paper submitted for publication to the International Journal of Nonlinear Mechanics [22]. We consider the system:

$$x'' + x - e(1 - ax^2 - bx'^2)\, x' = 0 \qquad (2.5)$$

which includes the Van der Pol and Rayleigh differential equations as special cases. Using Lindstedt's method and the Poincaré-Bendixson theorem, we show that for various values of the parameters a, b, e there exists a limit cycle in the x-x' phase plane. Using Poincare's technique of projecting the phase plane on the projective plane (i.e. a projection from the center of a sphere), we show that the limit cycle is created from four saddle-saddle connections between equilibrium points at infinity. Center manifold theory is used to determine the stability of the equilibrium points at infinity. Numerical

integration is used to verify the analytical results. Extensive use is made of MACSYMA and of the normal form program discussed in 1) above.

2.2.5 Computer algebra analysis for the stability of a rigid body with an oscillating particle (with L.A. Month). Paper submitted to the Journal of Applied Mechanics [23]. This work concerns the stability of the steady rotation of a rigid body containing a paricle which oscillates sinusoidally along a principal axis. The presence of the oscillating particle makes the criterion much more complicated than in the classical problem of the stability of a spinning rigid body (without an oscillating particle).

Mathematically, this problem involves the stability of an equilibrium position in a nonlinear dynamical system which is parametrically driven by a periodic forcing function. We offer a perturbation solution for transition curves in an appropriate parameter plane obtained by using MACSYMA. The use of computer algebra permits us to obtain expressions for the explicit dependence of the transition curves on the moments on inertia of the rigid body. This kind of result is in marked contrast to the traditional numerical approach to this kind of problem (in which one must assume numerical values for the moments of inertia at the outset.) In fact, we use such a numerical approach based on Floquet theory to confirm our computer algebra results.

2.2.6 Analysis of the slow-flow equations governing the first order perturbative solution to two coupled van der Pol oscillators (with T. Chakraborty). (Work in progress.) In a

1980 paper of Rand and Holmes, the problem of the dynamics of two linearly coupled Van der Pol oscillators was addressed. An approximate solution was derived in the form

$$x = R_1(t) \cos f_1(t) \;, \quad y = R_2(t) \cos f_2(t) \tag{2.6}$$

where the amplitudes R and the phases f depended on slow time t according to differential equations:

$$2R_1' + R_1(R_1^2/4-1) - aR_2 \sin f = 0$$

$$2R_2' + R_2(R_2^2/4-1) + aR_1 \sin f = 0 \tag{2.7}$$

$$-2f' + d + a(R_2/R_1-R_1/R_2) \cos f = 0$$

where $f = f_1 - f_2$ is the phase lag, and where a and d are parameters. We now continue the study of this problem by using normal forms and center manifold theory in conjunction with MACSYMA (see 1 above) to explore the dynamics of this system of equations. We show that the $a - d$ parameter plane contains a cusp catastrophe and a line of Hopf bifurcations, a sitatuion reminiscent of the associated problem of a single Van der Pol oscillator driven by a sinusoidal forcing function (investigated by Holmes and D. Rand in 1978). Current work involves the investigation of other bifurcations in these equations, including period-doubling.

2.3 Experiments on Chaotic Vibrations in a Feedback Controlled System, Dynamic Instabilities in a Rotating Manipulator Arm, and Parametric Stiffness Control of Space Structures (F.C. Moon, M. Golnaraghi (Grad Student), W.L. Keith)

2.3.1 Experimental Observations of Chaotic Vibrations in a Mechanical System with Servo-Motor Feedback Control.

To simulate the dynamical phenomena in a system of equations like (1.1) or (2.1), we have built a one degree of freedom linear positioning device (Figure 1). Magnets are used to create two prefered positions and the mass is programmed to oscillate between these two positions using a servomotor. A simple linear feedback law is used. The equations governing this electromechanical system are as follows:

$$\begin{aligned} m\ddot{x} + c\dot{x} - k_1 x + k_2 x^3 &= -F \ , \\ \dot{F} + \gamma F &= \beta\dot{x} + G(x - X(t)) \ , \end{aligned} \qquad (2.8)$$

where $X(t) = X(t + T)$ is a periodic reference function.

This system has been shown earlier [14] to exhibit chaotic dynamics in numerical simulation. These equations could represent an automated tool which is supposed to periodically switch from one work station to another. The object of our research is to see how fast the tool can follow the desired path without going chaotic.

The parameters in our experimental study are the switching time T and the feedback gain G .

Poincare maps of chaotic vibrations from the experiments are shown in Figure 2. When the motion is periodic, a finite set of dots are present. However, this Figure shows a large set of dots in sheet-like arrangements characteristic of strange attractors.

This work is continuing and will be reported in a paper in early Fall. We are planning to build a two or three degree of freedom device to simulate more complex robotic manipulators.

2.3.2 Dynamic Instabilities in a Two Degree of Freedom, Flexible, Rotating Manipulator Arm.

(Paper to be presented at a topical meeting on "Applications of MACSYMA" at the American Chemical Society meeting in Philadelphia, August 1984. In preparation for submission to *ASME Journal of Systems, Dynamics and Control*.)

In this work we have examined the dynamic stability of a two link manipulator with in-plane and out-of-plane flexibility (Figure 3). Using nonlinear and linear analysis, with the aid of MACSYMA, we have shown that both static and dynamic regions of instability exist (divergence and flutter) for given rotation rates and stiffness ratios (Figure 4). We have also run numerical simulation which show these local instabilities as well as the global motions. The equations for this manipulator arm fall into the general category equation given by (1.1).

<u>2.3.3 Parametric Stiffness Control of Flexible Structures</u>. (with R.H. Rand) Current proposals for the control of flexible structures often employ rockets or other force producing devices which will add considerable mass to the structure. We are exploring a new scheme where cables will be used to change the initial internal stress in the structure and thereby affect the stiffness matrix of the structure. In this theory we assume a structure expressed by mass and stiffness matrices, [20]:

$$[m]\{\ddot{x}\} + [m]\{x\} = f(t) \tag{2.9}$$

We assume that some elements of the stiffness matrix depend on various cable tensions T_k, i.e.,

$$k_{ij} = k_{ij}^{o} + \beta_{ijk} T_k \; . \tag{2.10}$$

In turn we assume that the cable tensions are controlled by a feedback control system and that these feedback loop obey equations of the form

$$\frac{1}{\alpha_k}\dot{T}_k + T_k = G_k(x_i, \dot{x}_j) \;, \tag{2.11}$$

where G_k are control gains and α_k are feedback lags.

We have shown that this system must employ a nonlinear control law in order to be globally stable. MACSYMA has been used along with normal form theory to analyze the stability of such systems (see section 2.2.2).

3. Proposed Research

In the main, we propose to continue our present lines of research. Some specific projects we envision are:

3.1 Analytical Studies of Three Dimensional Systems.

P.J. Holmes and S. Wiggins (Grad. Student) propose to expand their present study to include more general classes of three dimensional systems which, in some "unperturbed" form, become completely integrable. Perturbations of the Euler equations governing the motion of a rigid body in the absence of gravity or in free fall are a good example which has many applications in the study of sattelite dynamics. Melnikov methods are readily applicable to such problems (cf. Holmes and Marsden [24]), and our interest in them is prompted by a numerical study of chaotic motions in a feedback controlled rigid body problem (cf. Rand and Month's [1984] work outline in 2.2.5 above). More generally, we wish to develop tools for the study of the large amplitude (global) dynamics of a wider class of three dimensional systems which are "almost integrable". We also plan to consider periodically forced systems, an example of which is provided by the nonlinear oscillator studied by Holmes [14] in the presence of periodic position inputs to the feedback loop. See also §2.3.1 above and Figure 2. Numerical evidence due to Moon and Marzec and Spiegel [25] suggests that chaotic motions occur in both periodically forced and autonomous third order (feed back) systems, and we wish to obtain a better analytical understanding of these observations.

3.2 Analytical and Computer Algebra Analyses of Nonlinear Oscillators.

Work so far has used computer algebra (MACSYMA) normal form programs to deal with problems in which the degenerate singularity has a linearization of the form:

$$\begin{matrix} 0 & 1 \\ 0 & 0 \end{matrix} \quad \text{or} \quad \begin{matrix} 0 & -1 \\ 1 & 0 \end{matrix} \quad \text{or} \quad \begin{matrix} 0 & -w1 & 0 & 0 \\ w1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -w2 \\ 0 & 0 & w2 & 0 \end{matrix} \qquad (3.1)$$

In the case of the Van der Pol and Rayleigh equations at infinity, and in the case of a two mode model of a continuous structure with controlled stiffness, we found that while the MACSYMA programs could accomplish the transformation to normal form, the resulting nonlinear expressions were not determined, i.e., we could not use the existing theory of Takens [1974] to conclude that higher order terms would not change the topological nature of the truncated equations (Guckenheimer and Holmes, p. 360 et seq.). For example, in the case of the double zero eigenvalue, Takens shows that every such system may be reduced to the form (Guckenheimer and Holmes [17] p. 365):

$$x' = y + 0(3) \ , \quad y' = ax^2 + bxy + 0(3) \qquad (3.2)$$

where a and b are constants to be determined. However, in order for these equations to be 2-determined (i.e. in order to be

able to safely neglect terms of order 3), the constant a must be non-zero. In the case of the Van der Pol equation at infinity, however, we find ourselves in this precise situation, but where the constant $a = 0$.

In such a situation, we cannot use Takens' results, but we may hope to extend his results to cover these cases by using his method of "blowing up" the singularity (Guckenheimer and Holmes, pp. 362-364). The procedure involves a sequence of several (or possibly many) transformations, and involves laborous algebra. We propose to use MACSYMA to effect these transformations, with the goal of obtaining conditions for the determinacy of cases which were previously excluded from Takens' discussion.

In addition, we propose to continue our previous work on feedback control systems.

3.3 New Experiments in Dynamics of Feedback Controlled Elastic Systems

3.3.1 Chaotic vibrations. The experiments underway in the current grant deal with a one degree of freedom system (Figures 1 and 2). We propose to construct a manipulator arm with two or three degrees of freedom in order to explore the conditions under which the system will exhibit chaotic vibrations. We will also add elastic flexibility as a parameter as well as feedback gain in our exploration of strange attractor vibrations.

3.3.2 Measurement of fractal dimension of strange attractors. For systems with more than one degree of freedom it becomes difficult to experimentally measure Poincare mpas since we must project a high dimensional space onto a two dimensional place. Another tool which has been used to describe strange attractors is the concept of a fractal dimension. This idea is connected to the fact that a strange attractor in phase space does not have a dimension of integer value. The fractional dimension of the attractor provides a guide to the minimum number of degrees of freedom needed to describe the chaotic motion of the system.

We propose to develop an experimental technique coupled with computer data reduction which will enable us to measure the fractal dimension of strange attractor motions in experimental situations.

3.3.3 Experiments in stiffness controlled structures. We will attempt to initiate some exploratory studies of controlling flexible structures with tension controlled cables. These experiments will involve a flexible beam under cable controlled tension or compression. Given the major mathematical emphasis of this grant, only preliminary experimental studies will be initiated. We hope to explore the experimental parameter ranges in which the mathematical analysis of stiffness controlled structures should be carried out.

4. Reports and Papers Published or Written Under Present Grant

1. Holmes, P.J., "Bifurcation and Chaos in a Simple Feedback Control System," Proc. 22nd IEEE Conference on Decision and Control, San Antonio, TX, December 1983, Vol. 1, pp. 365-370.

2. Holmes, P.J., "Dynamics of a nonlinear oscillator with Feedback Control: I: Local Analysis," submitted for publication, 1984a.

3. Wiggins, S. and Holmes, P.J., "On Slowly Varying Oscillations," in preparation, 1984.

4. Holmes, P.J., "Knotted Orbits and Bifurcation Sequences in Periodically Forced Oscillations," XVIth ICTAM, Lyngby, Denmark, August 19-24, 1984b.

5. Moon, F.C., and Rand, R.H., "Parametric Stiffness Control of Flexible Structures," Proc. NASA Workshop on Identification and Control of Flexible Space Structures, San Diego, May 1984.

6. Keith, W.L. and Rand, R.H., "Dynamics of a System Exhibiting the Global Bifurcation of a Limit Cycle at Infinity," submitted for publication, 1984.

7. Moon, F.C., "Fractal Boundary for Chaos in a Two-State Mechanical Oscillator," Phys. Rev. Letters, January 1984.

8. Moon, F.C. and Li, G-X., "The Fractal Dimension of the Two-Well Potential Strange Attractor," accepted for presentation at XVIth International Congress of Theoretical and Applied Mechanics, Lyngby, Denmark, August 1984.

9. Moon, F.C., Keith, W.L., and Golnaraghi, M., "Instabilities in the Dynamics of a Two-Link, Flexible, Manipulator Arm," in preparation.

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14. Holmes, P.J., "Bifurcation and Chaos in a Simple Feedback Control System," Proc. 22nd IEEE Conference on Decision and Control, San Antonio, TX, December 1983, Vol. 1, pp. 365-370.

15. Holmes, P.J., "Dynamics of a nonlinear oscillator with Feedback Control: I: Local Analysis," submitted for publication, 1984a.

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17. Guckenheimer, J. and Holmes, P.J., "Nonlinear Oscillations, Dynamical Systems, and Bifurcations of Vector Fields," Springer Verlag, 1983.

18. Holmes, P.J. and Williams, R.F., "Knotted Periodic Orbits in Suspensions of Smale's Horseshow: Torus Knots and Bifurcation Sequences," in preparation 1984.

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20. Moon, F.C., and Rand, R.H., "Parametric Stiffness Control of Flexible Structures," Proc. NASA Workshop on Identification and Control of Flexible Space Structures, San Diego, May 1984.

21. Rand, R.H., "Hopf Bifurcation Via Lindstedt's Method" in preparation, 1984.

22. Keith, W.L. and Rand, R.H., "Dynamics of a System Exhibiting the Global Bifurcation of a Limit Cycle at Infinity," submitted for publication, 1984.

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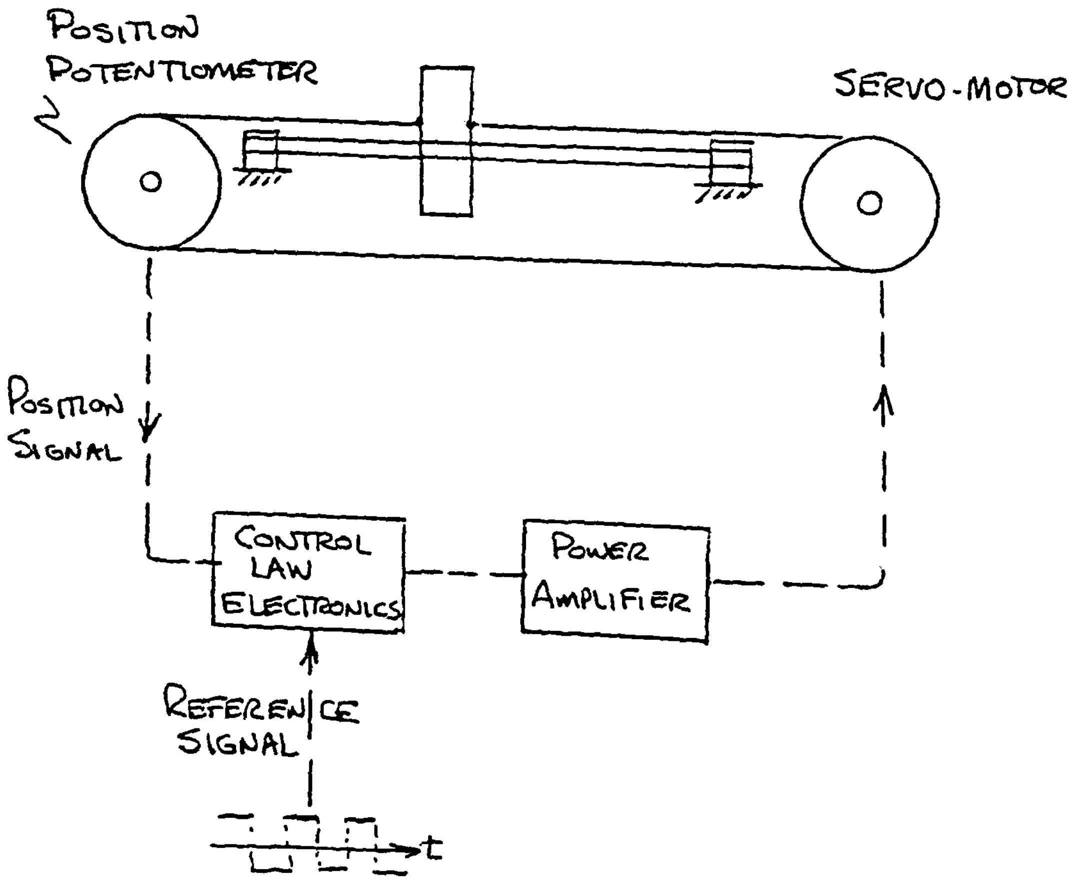


Figure 1: Experimental appartus for chaos in a feedback controlled system.

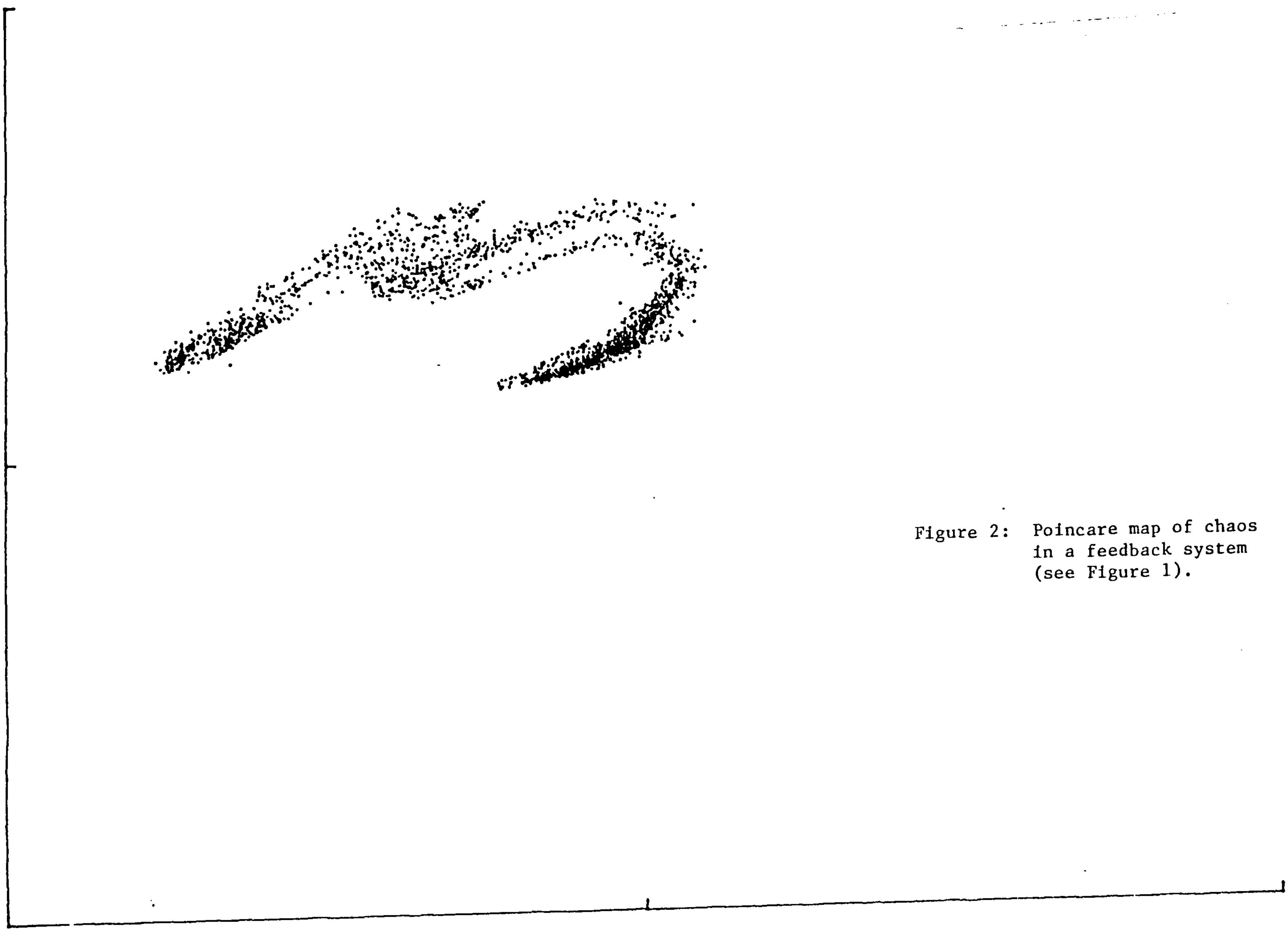
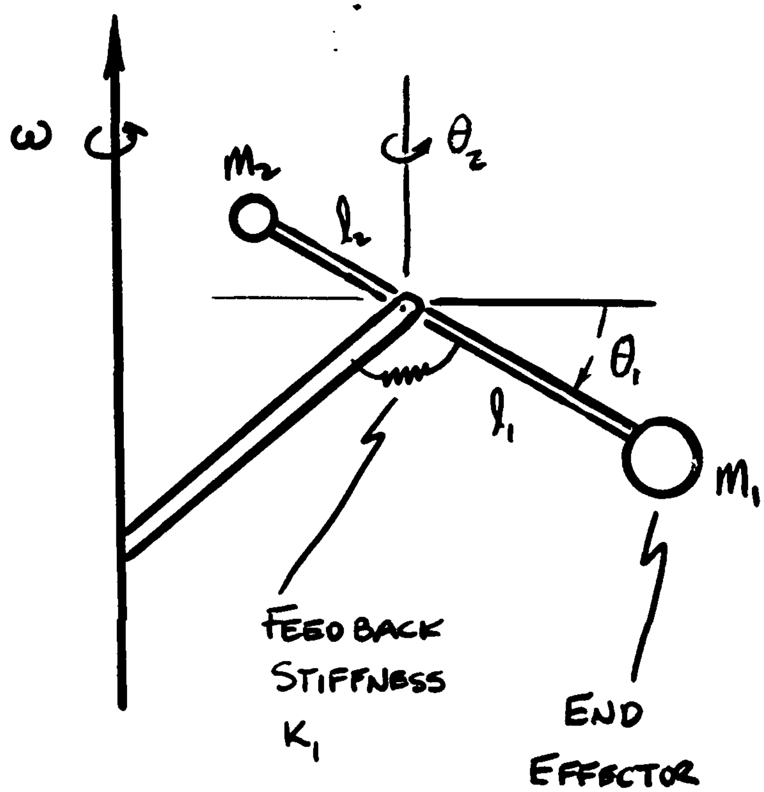
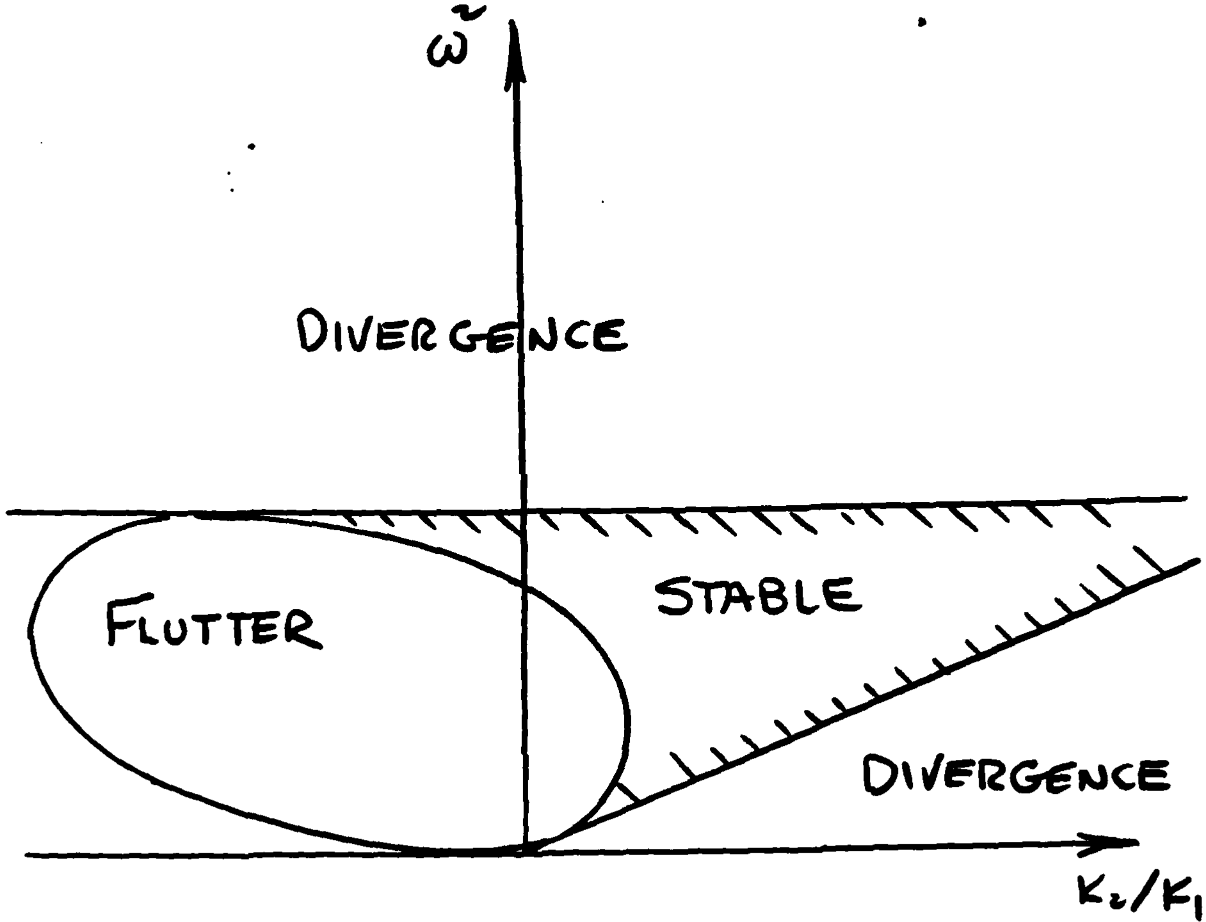


Figure 2: Poincare map of chaos in a feedback system (see Figure 1).



MANIPULATOR ARM
WITH IN-PLANE AND
OUT OF PLANE FLEXIBILITY (K_2)

FIG. 4



ω — ROTATION SPEED

K_2 — OUT OF PLANE STIFFNESS (ELASTIC JOINT)

K_1 - IN PLANE FEEDBACK STIFFNESS

List of Personnel

The following made contributions to this research program:

P. J. Holmes

Title: Professor

F. C. Moon

Title: Professor

R. H. Rand

Title: Professor

W. L. Keith

Title: Postdoctoral Associate

M. Golnaraghi

Title: Graduate Research Assistant

G-X Li

Title: Graduate Research Assistant

S. Wiggins

Title: Graduate Research Assistant

R. H. Rand

August 1984: American Chemical Society, Philadelphia, PA. Two papers presented at the topical symposium on MACSYMA.

June 1984: National Institute of Health Conference on Biomathematics (with Avis Cohen), "Lamprey Central Patten Generator."

W. L. Keith

August 1984: American Chemical Society, Philadelphia, PA. Two papers presented at the topical symposium on MACSYMA.

F. C. Moon

June 4-6, 1984: Lecture "Parametric Stiffness Control of Flexible Structures," (with R. H. Rand, Cornell University) NASA Workshop on Identification and Control of Flexible Space Structures, San Diego, Calif.

June 11-13, 1984: Participant in AFOSR sponsored Workshop on Space Structures.

August 13-17, 1984: Invited Lectures: " Eight lectures on experiments in chaotic vibration." Polish Academy of Sciences Symposium, Warsaw, Poland.

August 19-25, 1984: Lecture: "The Fractal Dimension of the Two-Well Potential Strange Attractor" (with G-X Li) XVIIth International Congress of Theoretical and Applied Mechanics, Lyngby, Denmark.

November 8, 1984: Invited Lecture: "Fractal Concepts in Chaotic Vibration in Mechanical Systems" University of Maryland, Department of Mathematics.

November 28-29, 1984: Invited Participant: Stanford Research Institute, NSF Workshop on New Directions in Solid Mechanics, Palo Alto, CA.

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